MA 437 Pre-Class Assignment for Chapter 3: Finite Groups; Subgroups — Fall 2014

Terminology and Notation.

1. Read Chapter 3, Terminology and Notation.
2. What is the difference between a finite group and an infinite group?
3. Write the definition of the *order of a group*.
4. What is the order of the group $\mathbb{R}^*$, the nonzero real numbers under multiplication? What is the order of the group $U(18)$?
5. Write the definition of the *order of an element* in a group.
6. Study the computations in Examples 1, 2, and 3.
7. Do Exercise 1.
8. Write the definition of a *subgroup* of a group.
9. It's worth noting here that since the definition of a group implies the existence of an identity element, a group can never be empty. In particular, subgroups of groups are never empty.
10. Is $\mathrm{GL}(2, \mathbb{Q})$ a subgroup of $\mathrm{GL}(2, \mathbb{R})$? Is the group $\mathbb{Z}_n$ under addition modulo n a subgroup of $\mathbb{Z}$ under addition?
11. What is a *proper* subgroup of a group? What is the *trivial* subgroup?
12. What notation is used to mean that H is a subgroup of G? Is there a special notation used when H is *proper* subgroup of G?
13. Let $H \leq G$. Prove that the identity of H is equal to the identity of G. (Hint: Show that the identity of G is an element of H, then use uniqueness of the identity.)
14. Do Exercise 4. You may find the following steps helpful.
    (a) Let $a \in G$. Let $|a| = n$ and $|a^{-1}| = k$. For now, assume that n and k are finite.
    (b) Show that $k \leq n$ by computing $(a^{-1})^n$.
    (c) In the same spirit, show that $n \leq k$ by computing a^k. It may help to notice that $a = (a^{-1})^{-1}$.
    (d) What do the results of (a) and (b) imply about $|a|$ and $|a^{-1}|$?
    (e) Does the result Exercise 4 still hold if a has infinite order?