MA 431 – Definition of a Vector Space **Fall 2014**

Definition. Let F be a field (In our class, $F = \mathbb{R}$ or $F = \mathbb{C}$). The elements of F are called *scalars*. A *vector space* over F is a set V (the elements of V are called *vectors*), together with two operations (called *addition* and *scalar multiplication* such that

- for all vectors $x, y \in V$, there is a unique vector $x + y \in V$ (called the *sum* of x and y), and
- for all vectors $x \in V$ and for all scalars $a \in F$, there is a unique vector $ax \in V$ (called the *product* of a and x),

and such that the following eight properties hold.

(VS 1) For all vectors $x, y \in V$, $x + y = y + x$. (Vector addition is commutative.)

(VS 2) For all vectors $x, y, z \in V$, $(x + y) + z = x + (y + z)$. (Vector addition is associative.)

(VS 3) There exists a vector $\vec{0} \in V$ (called the *zero vector*) such that $x + \vec{0} = x$ for all vectors $x \in V$.

(VS 4) For all vectors $x \in V$, there exists a vector $y \in V$ such that $x + y = \vec{0}$. (The vector y is often denoted $-x$.)

(VS 5) For all vectors $x \in V$, $1x = x$ (where $1 \in F$ is the scalar 1).

(VS 6) For all scalars $a, b \in F$ and all vectors $x \in V$, $(ab)x = a(bx)$.

(VS 7) For all scalars $a \in F$ and all vectors $x, y \in V$, $a(x + y) = ax + ay$.

(VS 8) For all scalars $a, b \in F$ and all vectors $x \in V$, $(a + b)x = ax + bx$.