MA 431 – Comments for Homework 1 Spring 2015

Guidelines for rewrites.

- Every student is permitted *one rewrite per homework assignment*. All problems to be rewritten must be submitted at the same time, *together with the original submission*.
- Rewrites are due *two class days after return of the original homework assignment*.
- In order to submit a rewritten problem, you must submit a new solution *to the entire problem*, not just the parts you wish to rewrite.
- Instructor comments on your original submission are not intended to correct every mistake. Some errors in your original solutions may not be indicated by instructor comments. It is your job to revisit (or scrap) your *entire* original solution to ensure that your rewritten solution is correct.
- If some comments on your original solution are unclear, just ask the instructor for clarification!

Comments on Homework 1.

- It is important to use the $\vec{0}$ notation when referring to the zero vector in a vector space. This is to distinguish the zero vector from the zero scalar, 0. This mistake was especially prevalent in Problem 9, and this creates a good amount of confusion for the reader (which should be avoided at all costs!).
- In Problem 12, let V be the set of even functions $\mathbb{R} \to \mathbb{R}$. We wish to show that V is a vector space over $\mathbb{R}$. Before you dive right into showing that (VS 1)–(VS 8) hold for V under the usual operations of addition and scalar multiplication of functions, we need to show that if $f, g \in V$, then $f + g \in V$, and that if $f \in V$ and $a \in \mathbb{R}$, then $af \in V$. This is the first part of the definition of a vector space! This same part of the definition of a vector space was overlooked in Problem 15.
- In general, the most transparent way to show that some statement is *not true* is to exhibit a counterexample to the statement. For example, suppose we are given a set V equipped with an addition and a scalar multiplication, and we suspect that one of the properties of a vector space, say (VS 1), fails. In order to *show* that (VS 1) fails, very little generality is needed. Just find *specific* vectors $x, y \in V$ where $x+y \neq y+x$. An example counterexample demonstration appears below.
  - **Problem.** Let $V = \{(a_1, a_2) \mid a_1, a_2 \in \mathbb{R}\}$ with addition defined by $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 - b_2)$ for all $(a_1, a_2) \in V$, and scalar multiplication defined by $c(a_1, a_2) = (ca_1, ca_2)$ for all $c \in \mathbb{R}$. Is V a vector space over $\mathbb{R}$?

    Solution: No, V is not a vector space over $\mathbb{R}$. Indeed, consider $(1, 2), (3, 4) \in V$. Observe that

    $$(1, 2) + (3, 4) = (1 + 3, 2 - 4) = (4, -2),$$

    but

    $$(3, 4) + (1, 2) = (3 + 1, 4 - 2) = (4, 2).$$

    Hence, since $(1, 2) + (3, 4) \neq (3, 4) + (1, 2)$, (VS 1) fails, so V is not a vector space.