Math 110 Supplemental Instruction Worksheet 5

1. A club of 12 people, made up of 4 Math Majors, 3 Computer Science Majors and 5 Physics Majors, are selecting members for a four person committee.

   (a) How many ways are there to select four people for this committee?

   $$C(12,4) = \frac{12!}{8!\,4!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{11880}{24} = 495$$

   (b) How many ways are there to select one person to be president of the committee, and an additional three people for members?

   $$C(12,1) \cdot C(11,3) = 12 \times \frac{11!}{8!\,3!} = 12 \cdot \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1}$$

   one pres   3 others.

   $$= 12 \cdot \frac{990}{6}$$
   $$= 12 \cdot 165$$
   $$= 1980$$

   (c) How many ways are there to select four people for this committee if there must be exactly one math major on it?

   $$C(4,1) \cdot C(8,3) = 4 \cdot \frac{8!}{5!\,3!} = 4 \cdot \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$$

   one math   3 not math

   $$= 4 \cdot 56$$
   $$= 224$$

(d) How many ways are there to select four people for this committee if we must have at least one student from each major? (Hint: What are your all your options if you have four people, and must have one from each major?)

$$2M, 1CS, 1P \quad \text{or} \quad 1M, 2CS, 1P \quad \text{or} \quad 1M, 1C, 2P$$

$$C(4,2) \cdot C(3,1) \cdot C(5,1) + C(4,1) \cdot C(3,2) C(5,1) + C(4,1) \cdot C(3,1) \cdot C(5,2)$$

$$= \frac{4!}{2!2!} \cdot \frac{3!}{2!1!} \cdot \frac{5!}{4!1!} + \frac{4!}{3!1!} \cdot \frac{3!}{1!2!} \cdot \frac{5!}{4!1!} + \frac{4!}{3!1!} \cdot \frac{3!}{2!1!} \cdot \frac{5!}{3!2!}$$

$$= 6 \cdot 3 \cdot 5 + 4 \cdot 3 \cdot 5 + 4 \cdot 3 \cdot 10 = 90 + 60 + 120 = 270$$

2. In how many different ways can First, Second and Third place be awarded in a bike race with 15 contestants?

$$P(15,3) = 15 \cdot 14 \cdot 13 = 2730$$

3. On a counter, you have four different hair products, two different nail products and three different medicines. How many ways are there to arrange these products in a row if each type of product needs to be grouped together? Like couches problem!

Arrange hair products: $4! = 24$
Arrange nail products: $2\cancel{8}! = 2$
Arrange medicines: $3! = 6$
Arrange groups: $3! = 6$

$$24 \cdot 2 \cdot 6 \cdot 6 = 1728$$