Name: ____________________

Probability Review

Make a Sample Space.

If you're going to use a sample space to calculate the probability of an event E, then you simply count:

$$P(E) = \frac{n(E)}{n(S)}$$

- If you use a sample space, all outcomes in your sample space must be **equally likely**.

  Example: If you are finding the probability of drawing two kings from a deck of 52 cards, then the following is NOT a correct sample space

  $$\{(\text{King, King}), (\text{King}, \text{Not King}), (\text{Not King}, \text{King}), (\text{not King, not King})\}$$

  because a king is much less likely than a card that is not a king. If you wanted to make a sample space, then you would have to have pairs of every card. I don't recommend that!

  Example: A box contains 3 yellow, 5 red and 6 blue marbles. Find the probability that you draw two blue. The following is NOT a correct sample space:

  $$\{(Y, Y), (Y, R), (Y, B), (R, Y), (R, R), (R, B), (B, Y), (B, R), (B, B)\}$$

  because the probability of drawing a blue marble is twice the probability of drawing a yellow marble! Here you could use counting, or conditional probability (probability tree).

- Good Example:

  Question: You flip a coin three times. What is the probability that you get at least 1 head?

  Solution: Sample Space = $\{(h, h, h), (h, h, t), (h, t, h), (t, h, h), (t, t, h), (t, h, t), (h, t, t), (t, t, t)\}$.

  So the probability is

  $$P(\text{at least one head}) = \frac{n(\text{at least one head})}{n(\text{sample space})} = \frac{7}{8}$$

Make a probability tree.

If you're going doing something one time and then another, where the second outcome depends on what the first outcome was, then you can make a probability tree.

Question: A box contains 3 yellow, 5 red and 6 blue marbles. Find the probability that you draw a yellow and a blue marble.

Answer: Here you could draw a probability tree. The first tier has three branches, one for each color, and the second has three branches, one for each color (with the probabilities of the second tier depending on the first.

Tip: Look for the words "Given" or "of those". That's a tip off that you'll want a probability tree:

Question: In a particular elementary school, 27% of students want to be astronauts. Of those that want to be astronauts, 52% doesn't know what an astronaut does, while 71% of those who do not want to be an astronaut don't know what an astronaut does. What is the probability that a given child both wants to be an astronaut and knows what an astronaut does.

Answer:

Count Things.

Use counting when you're drawing/choosing/selecting things without replacement.

Question: A box contains 3 yellow, 5 red and 6 blue marbles. Find the probability that you draw a yellow and a blue marble.

Answer: Here you could also use counting! If we're choosing 2 marbles out of a total of $3+5+6=14$ marbles, then our denominator is $\binom{14}{2}$. Since we're wanting to choose one yellow and one blue, the top is $\binom{3}{1}$ for the yellow, times $\binom{6}{1}$ for the blue. Thus we get

$$P(\text{one yellow and one blue}) = \frac{\binom{3}{1}\binom{6}{1}}{\binom{14}{2}} = \frac{18}{91}$$

You can usually use counting when you have a problem that *looks* binomial (repeated trials, focusing on one outcome) but whose probability is not constant from trial to trial.

Remember to ask yourself: "How many do I want?" and "How many are there?".

Use the Binomial formula.

If you have a situation where you are doing the same experiment repeatedly, counting the number of times a particular outcome occurs and *the probability the desired outcome occurs does* ***not*** *change from trial to trial*, then you have a Binomial probability problem.

Question: A coin is flipped 20 times. What is the probability that heads comes up 18 times?

Answer: Since we are flipping a coin, counting the number of heads and the probability of heads does not change each coin flip (always $\frac{1}{2}$), we have a binomial probability problem. In this problem, $n = 20$, the number of trials of our experiment, $x = 18$, the number of times we want our desired outcome, and $p = \dfrac{1}{2}$, the probability that the coin reads heads on any given flip. So the probability is

$$\binom{n}{x}p^x(1-p)^{n-x} = \binom{20}{18}\left(\frac{1}{2}\right)^{18}\left(1-\frac{1}{2}\right)^{2} = .000181$$

Question: A flu vaccine has a probability 80% of preventing a person who is inoculated from getting the flu. A county health office inoculates 15 people. Find the probability that exactly 4 of the people inoculated get the flu.

Answer: We are checking people for the flu and counting the number of people who got the flu after getting a flu shot. Since the probability I get the flu after getting the shot does not affect the probability that you get the flu after getting a flu shot, this is binomial probability. In this problem, $n = 15$, $x = 4$ (the number of people who *do* get the flu after getting a flu shot) and $p = 1 - .8 = .2$. Note that we have to subtract the probability from 1, since 80% is the probability a person does not get the flu and we are counting the people who do get the flu. Thus we have the probability is:

$$\binom{15}{4}(.2)^4(1-.2)^{11} = .188$$

Often times when a probability problem involving people that we are questioning, not choosing, it is binomial. Not always, but often.